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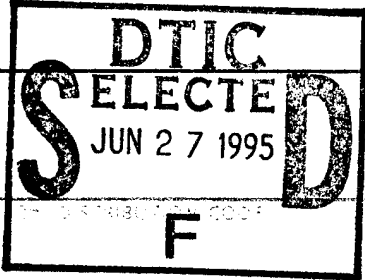
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Moment closures were developed for modeling the transition regime in kinetic theory. This was done using a systematic nonperturbative derivation of a hierarchy of closed systems of moment equations. Criteria were developed regarding the stability of shocks in magnetohydrodynamics. Finally, the new insight was gained into the diffraction effects that occur when a weak shock interacts with a sharp wedge, the von Neumann paradox.

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Executive Summary

We have made a major breakthrough in the development of moment closures for modeling the transistion regime in kinetic theory. Specifically, we have found a systematic *nonperturbative* derivation of a hierarchy of closed systems of moment equations corresponding to any classical kinetic theory. The first member of the hierarchy is the Euler system, which is based on Maxwellian velocity distributions, while the second member is based on non-isotropic Gaussian velocity distributions. The closure proceeds in two steps. The first ensures that every member of the hierarchy is hyperbolic, has an entropy, and formally recovers the Euler limit — fundamental properties that are lacking in previous closures. The second involves modifying the collisional terms so that members of the hierarchy beyond the second also recover the correct Navier-Stokes behavior. The simplest such system in three spatial dimensions recovers the behavior of the Grad "13-moment" system when the velocity distributions lie near local Maxwellians. The closure procedure can be applied to a general class of kinetic theories. There are a number of ongoing foolow-up projects aimed at a practical exploitation of this advance.

In addition, we have made advances in understanding the foundations of fluid dynamics, identified distinguished weakly compressible limits, developed intrinsic criteria for checking the validity of compressible Navier-Stokes simulations, and many other contributions to the theory of compressible fluid dynamics.

We have also gained new insights into the stability of shocks in MHD, which comes into play at high Mach numbers. We have showed using numerical experiments that when the ratio of the viscosities is below a critical value, intermediate viscous profiles exist and their stability depends on the relative sizes of the viscosity coefficients and the size and the type of the perturbation applied.

On another topic pertinent to the grant, the numerical study of diffraction effects for the interaction of a weak shock with a sharp wedge we concentrated on the resolution of the so-called von Neumann paradox. Various scenarios have been tried over the years (since the late forties), but they did not resolve the problem. Our numerical solution seems to support the conjecture recently put forward by Collela and Henderson that there is a fourth wave. Our results show that the strength of this wave is proportional to the curvature of the stem. Since this curvature is very small, it was impossible to detect in previous physical and numerical experiments due to the lack of sufficient resolution.

1. Research Objectives

The proper simulation of rarefied gases presents a formidable challenge to computational science. In situations where the gas is sufficiently dense that the mean free path of a particle (the average distance traveled between collisions) is much smaller than the macroscopic length scales of interest, fluid dynamical descriptions are valid. In that case the particles will approach a local equilibrium parameterized by so-called fluid variables (typically the mass density, fluid velocity, and temperature) whose evolution will be governed either by the compressible Euler equations, which approximate the velocity distribution by a local equilibrium, or the compressible Navier-Stokes equations, which account for small deviations of the velocity distribution from a local equilibrium. Such fluid dynamical equations are routinely solved numerically to effectively model gases in a wide variety of applications.

However, when a gas is sufficiently rarefied that the mean free path is no longer much smaller than the macroscopic length scales, the deviation of the velocity distribution from a local equilibrium may become large and Navier-Stokes equations can yield momentum and energy fluxes that are inconsistent with nonnegative distribution functions and that may even be wrong by orders of magnitude. In that case the gas can be modeled by a kinetic theory of dilute gases like that governed by the Boltzmann equation. The gas is then described by single particle phase-space densities (one for each species) rather than fluid dynamical variables and the evolution of these phase-space densities is then governed by kinetic equations. Such equations may be effectively solved via molecular dynamics or Monte-Carlo methods at low densities that are far outside fluid dynamical regime. However, because of its phase-space description and numerical stiffness, the computational cost of doing so in regimes near the fluid dynamical limit becomes too prohibitive in both time and storage requirements to allow for general usage. An alternative to a full kinetic description is to use a moment closure, but these too are significantly more expensive than a Navier-Stokes simulation.

The gap in our ability to efficiently model gases in the regime that lies between free molecular flow and fluid dynamics lies in what is called the transition regime. One central objective of this work has been to develop models that fail gracefully as one leaves the fluid dynamical regime. More precisely, we seek models that properly capture the fluid dynamical regime when the mean free

path is much smaller than the macroscopic length scales, while in the transition regime give values for the momentum and energy fluxes (and other quantities) that are at least consistent with the nonnegativity of the particle density, and are thereby hopefully of the correct order of magnitude. By doing so, such models may provide a bridge over the transition regime that may be useful in the construction of hybrid fluid/kinetic simulations.

More specifically, the intent of our work has been to explore alternative strategies to the Chapman-Enskog procedure in order to develop a generalized hydrodynamics. A few criteria have to be met:

1. These equations should meaningfully extend into regimes beyond that of the Navier-Stokes equations.

2. Their mathematical structure must be robust, free from unphysical anomalies.

3. They should have a reasonable complexity; if this is not the case then one might as well be resigned to using the original Boltzmann equation.

Two such strategies have been examined: the development of flux-limited Navier-Stokes equations and, more extensively, the closure of moment equations. The long term goal is to make hybrid fluid/kinetic calculations more tractable.

At Mach numbers near ten ionization becomes important. We therefore examined problems related to asymptotic and numerical studies of a model MHD equation with high order effects included. The model describes asymptotically a uni-directional MHD wave propagation near the point where three characteristic speeds coincide and may include weak dissipative, dispersive and diffractive effects. The main advantage of our model is in its simplicity while preserving the essential wave characteristics of the MHD equations.

The aim of this work was to study computationally wave propagation and interaction for problems in magnetohydrodynamics described by our model equation. The immediate goal is to study properties of singular solutions for asymptotic equations, namely, intermediate shock wave stability, resistive and dispersive perturbations of the Alfvèn waves and diffraction of shock waves. The last topic is a computational study of interaction of weak shock with the sharp wedge by using an ENO-type scheme adopted to deal with the diffraction "wiggles".

2. Status of the Research

2.1 Fluid Dynamics: Validity and Breakdown (Levermore)

To establish the validity of fluid dynamics from an underlying Boltzmann equation has been considered one of the classical open problems of mathematical physics for over one hundred years. Of course, formal derivations exist for classical fluid dynamical approximations, but that is not the same as a mathematically rigorous proof. Such proofs would have great value in helping to understand the accuracy of numerical schemes for solving the Boltzmann equation. Bardos, Golse, and Levermore [5] have made a major step in this direction for the case where the fluid limit is that of a viscous incompressible fluid. Their starting point was the recent theory of global weak solutions of the Boltzmann equation due to DiPerna and Lions; their goal was therefore the theory of global weak solutions of the Navier-Stokes equations due to Leray in 1933. They considered the classical Boltzmann equation over a periodic spatial domain for a wide class of Boltzmann collision kernels.

By using relative entropy estimates about an absolute Maxwellian, it was shown that any properly scaled sequence of DiPerna-Lions renormalized solutions of some classical Boltzmann equations has fluctuations that converge to an infinitesimal Maxwellian with fluid variables that satisfy the incompressibility and Boussinesq relations globally in time. Moreover, they introduced a notion of entropic convergence and showed that if the initial fluctuations entropically converge to an infinitesimal Maxwellian then the limiting fluid variables were shown to satisfy a version of the Leray energy inequality for the Navier-Stokes, again, globally in time [5].

By assuming the sequence satisfies local momentum conservation, the appropriately scaled momentum densities globally are shown to converge strongly to the solution of the Stokes equation. A similar discrete time version of this result holds for the Navier-Stokes limit with an additional mild weak compactness assumption [5].

The utility of the concept of entropic convergence was demonstrated by Levermore in [6]. There it was first shown, again using relative entropy estimates about an absolute Maxwellian, that any properly scaled sequence of DiPerna-Lions renormalized solutions has fluctuations that converge weakly to

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a solution of the linearized Boltzmann equation. Then it was shown that, whenever the initial fluctuations converge entropically to an arbitrary L^2 initial data of the linearized Boltzmann equation, the DiPerna-Lions fluctuations converge entropically (and hence strongly in L^1) to the unique L^2 solution of the linearized Boltzmann equation with the given initial data. In a sense, this is an infinitesimal uniqueness result for DiPerna-Lions solutions.

The same scalings that led to incompressible fluid dynamics from the Boltzmann equation in [5] where applied in [1] by Bayly, Levermore, and Passot to derive a theory of turbulence in weakly compressible fluid flows. In that work, two distinguished scalings were found that led to different relations between the fluctuations of the density and the velocity fields. One of these had been missed in some earlier literature which had based its theory on the isentropic approximation. These relations were illustrated with careful simulations of the full compressible Navier-Stokes equations.

As nice as the preceding results are, they are far from what we need for practical aerospace applications, where the fluid considered is usually very compressible. Levermore, Morokoff, and Nadiga therefore developed a criteria for monitoring the validity of the compressible Navier-Stokes approximation during the simulation of a rarefied gas [24]. Because the Navier-Stokes equations can be systematically recovered from the underlying kinetic description in the regime of small mean free paths by truncating either a Hilbert or Chapman-Enskog expansion, kinetic theory can provide the foundation for any such criterion. For example, one could try to use the first neglected term in the Chapman-Enskog expansion to estimate the error. That was not the approach they took. Rather, they found a criteria based on the requirement that certain velocity moments generated by the velocity distribution of the truncated Chapman-Enskog expansion (which is generally not nonnegative) be realizable by some nonnegative velocity distribution. Such criteria are called *moment realizability criteria*. They then generalized this approach to a more sensitive criterion which measures the deviations of the underlying distribution function from equilibrium. One important aspect of these criteria is that they involve quantities that are intrinsic to the Navier-Stokes equations. This method, based on the deviations of a 3×3 validity matrix from its equilibrium value of the identity, is portable and may be applied to any Navier-Stokes simulation. They examined its utility by comparing stationary planar shock profiles computed using the Navier-Stokes equations with those computed using Monte-Carlo simulations [24].

Once one has left the regime of validity for the compressible Navier-Stokes equations there are a number of things one might do, one of which is the de-

velopment of flux-limited Navier-Stokes equations. The key idea here is to base the fluid dynamical closure on a family of approximate solutions that can deviate considerably from local equilibria. For example, the closure of flux-limited diffusion theory can be viewed as choosing the distribution corresponding to a traveling wave solution of the transport equation which is consistent with the values of both the particle density and its gradient. In the fluid dynamic context this means constructing leading order particle distributions not only from the fluid dynamic variables (density, velocity, and temperature), but also from their gradients. If this is to be done in a physically consistent manner, attention must be paid to the implied entropy dissipation rate. Even for the classical compressible Navier-Stokes equations, consideration of the entropy dissipation rate tells us that it is velocity and temperature, rather than other variables, which are diffused. These variables are identified by being dual to the conserved densities (mass, momentum, and energy) with respect to the entropy. If the underlying particle distribution now depends functionally (rather than algebraically) on the conserved densities then so will the entropy and the dual variables associated with it.

Levermore and Wagner [8] have applied these ideas to the simple Broadwell model. For this model the flux-limited closure can be checked against benchmark calculations of the full kinetic equation, something that is completely out of the question for more realistic kinetic theories. The results looked promising but the true test will come when these ideas are applied to more realistic kinetic equations.

2.2 Moment Closures (Levermore)

Another strategy to describe deviations from fluid dynamics is to close systems of moment equations such as typified by the thirteen moment closure of Grad. This strategy introduces dynamical equations for velocity moments of the particle distribution beyond those of the conserved mass, momentum, and energy densities. The additional equations are not local conservation laws, but rather local relaxation laws that include moments of the collision operator. Any closure of this enlarged system must approximate both the higher flux moments and the collision operator moments. These closures often use relations that are only justified when the particle distribution is near a local equilibrium, such as Grad's moment truncation of the fluxes in terms of generalized Hermite polynomials relative to the local Maxwellian, and his so-called "diagonal approximation" of the collision operator. The resulting systems of equations involve shorter spatial-temporal scales but retain the assumption of closeness to the local equilibria, thereby taking on a perturbative nature.

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Such systems of moment equations present many problems that must be faced if they are to be useful tools for simulation in the transition regime. The most significant of these problems are:

(1) complexity due to the large number of equations,

(2) stiffness near the fluid dynamical limit,

(3) loss of realizability of its predicted moments,

(4) breakdown away from moderate regimes.

The first problem is intrinsic to this general strategy and carries a substantial computational cost for any simulation. This problem is being mitigated by advances in supercomputers. The second is also intrinsic but Jin and Levermore [21] showed that this problem can be resolved by a proper choice of numerical scheme. The third problem is more serious because it means that the predicted values of the moments can evolve to the point where they violate inequalities that must be satisfied if they are to be realized by *any* nonnegative density. This problem can be monitored by checking that the predicted moments satisfy appropriate moment realizability inequalities during a simulation. The last problem arises because such systems can dynamically become elliptic (develop complex characteristics) and hence become ill-posed, after which the meaning of the solution becomes suspect.

In a major new work, Levermore [23] has given a systematic *nonpertur-bative* derivation of a whole hierarchy of closed systems of moment equations corresponding to any classical kinetic theory. The first member of the hierarchy is the Euler system, which is based on Maxwellian velocity distributions, while the second closure is based on non-isotropic Gaussian velocity distributions. The closure procedure has two steps. The first ensures that every member of the hierarchy is hyperbolic, has an entropy, and possesses realizability of its predicted moments, thus ensuring that, unlike the perturbative approach, difficulties such as (3) and (4) above do not arise. Moreover, every member formally recovers the Euler limit. The second step involves a modification of the collisional terms that is a nonlinear generalization of the "diagonal approximation" of Grad and which ensures and those members of the hierarchy beyond the Gaussian closure recover the correct Navier-Stokes behavior. The simplest such system in three spatial dimensions is a "14-moment" closure which also recovers the Grad "13-moment" system when the velocity distributions lie near local Maxwellians.

Although simple, the Gaussian closure does not recover the correct Navier-Stokes approximation because the Gaussian densities have no heat flux, and

therefore no heat conduction term can arise in the energy equation. However, all members of the hierarchy above the Gaussian closure have a nontrivial heat flux and, hence, hold out the possibility that the correct Navier-Stokes approximation can be recovered as the first correction to the Euler equations. A general theory of such approximations was worked out by Chen, Levermore, and Liu in [9]. Following that work, it can be shown that all such higher order closures lead to the correct form of the Navier-Stokes stress and heat flux [23]. For realistic collision operators the values of the viscosity and heat conduction derived from such closures will generally be less than the correct physical values. However, it was shown that the collision operator can be modified so as to recover the correct physical viscosity and heat conduction by introducing a generalized BGK model tuned so as to recover various relaxation times correctly [23]. This is the nonlinear analog of the diagonal approximation of Grad, and it even reduces to the Grad approximation near a local equilibrium.

The main difficulty in implementing this moment closure lies in the complexity of its equation of state. For this reason, it is hoped that one might be able to correct the Gaussian closure, which has no such complexity problem. This is the impetus for the works of Groth and Levermore [43] and Levermore and Morokoff [44]. The first finds transport corrections to the Gaussian Closure based on an asymptotic expansion, while the second examines the attendant Riemann problem. There is still much work left to be done.

This work has, before its publication, attracted the attention of researchers a round the world. Derivative work has already been submitted to journals from the University of Michigan group of Gambosi, Groth, Roe, and Brown. Other work is forthcoming out of Bordeaux, France by a group led by Charrier.

Finally, in related work, Harten, Lax, Levermore, and Morokoff [22] have extended determination of which entropy densities for the compressible Euler equations of the form $\rho f(\sigma)$ are strictly convex (where ρ is the mass density, σ is the specific entropy, and f is an arbitrary function) from polytropic gases to gases with an arbitrary equation of state. Moreover, they showed that at every state where the sound speed is positive (i.e. where the Euler equations are hyperbolic) there exist $\rho f(\sigma)$ that are strictly convex, thereby establishing the converse of the general fact that the existence of a strictly convex entropy density implies hyperbolicity.

2.3 MHD Model Equations (Brio)

This work pertinent to the proposal concentrated on computational and asymptotic studies of the MHD model equations.

In Brio and Rosenau [30, 41], and Cheng, Brio and Webb [10] the model system describing unidirectional wave propagation was used to address the question of whether intermediate shock waves appear in nature. This question goes back to the beginning of MHD studies in the mid-50s, but has not yet been resolved satisfactorily. More specifically, we have studied numerically the nonlinear stability of intermediate viscous profiles. The numerical experiments show that when the ratio of the viscosities is below a critical value, intermediate viscous profiles exist and their stability depends on the relative sizes of the viscosity coefficients and the size and the type of the perturbation applied. In particular, for fixed viscosity coefficients only one type of intermediate shock can be split by a sufficiently large fast or slow perturbation for the coplanar 2×2 model system and the rest of the intermediate shocks are stable. The 2×2 model is useful for 2D MHD simulations. In the 3×3 case, all intermediate shocks can be destroyed by a sufficiently large rotational perturbation. Our numerical results indicate the following effect of dispersion (the screw-symmetric part of the viscosity matrix due to the Hall effect). If the diffusion is small enough, the dispersive effects may lead to the formation of rotational Alfvénic perturbations and to the break up of the intermediate waves.

Another important observation is that an Alfvén wave under small viscosity perturbation is transformed into a nearby intermediate wave, which, for example, may consist of a shock followed by a rarefaction wave of the same family. This is drastically different from a contact discontinuity in the hydrodynamic case, which spreads out as $\sqrt{t}$ in the presence of small viscosity. Here the spread may be on the advection scale and may account for the difficulty in observing Alfvén waves in a resistive medium.

Our numerical simulations suggest an explanation of the previous experimental observations and numerical results. For example, it suggests that in the magnetosphere, intermediate shocks may be observed very rarely, since the resistivity in space plasmas is small compared to the Hall effect, and large Alfvén (rotational) waves are abundant in interplanetary space. For numerical computations, it provides an explanation of why the leap-frog and Lax-Wendroff type schemes, when used for 2D computations, contain rotational (Alfvén) waves, while modern upwind approximation methods (like TVD, PPM, and ENO schemes) converge to intermediate waves instead.

Encouraged by our current and previous numerical results, several other researchers reported experimental evidence for intermediate shock waves that were previously disregarded as unphysical and unstable mathematical solutions unrelated to physical reality. The results opened the door to further investigations, applications, and guidelines for development of numerical schemes. In our most recent work on this topic, we extend our study in order to include the effects of dispersion on the intermediate shocks and Alfven waves [26, 46].

2.4 Diffraction Effects (Brio)

The second topic pertinent to the grant, is a numerical study of diffraction effects. In Brio and Hunter [2, 4, 25, 36], using the 2D Burgers equation we studied the problem of the interaction of a weak shock with a sharp wedge. In particular, we concentrated on the resolution of the so-called von Neumann paradox for weak shock Mach reflections. The paradox stems from the analytical result that there is no solution to the problem of three shocks meeting at a point, whereas numerical and experimental evidence seems to contradict it. Various scenarios have been tried over the years (since the late forties), but they did not resolve the problem.

Our numerical solution seems to support the conjecture recently put forward by Collela and Henderson that there is a fourth wave. Our results show that the strength of this wave is proportional to the curvature of the stem. Since this curvature is very small, it was impossible to detect in previous physical and numerical experiments due to the lack of sufficient resolution. The computation is very delicate — the diffraction acts "dispersively" by introducing wiggles that persist under numerical viscosity in the ENO method, since the nonlinearity in the equation is in x-direction only (diffraction is a linear global effect in the y-direction). The addition of linear artificial viscosity in the y-direction smears the diffractive wave and is unsatisfactory. To remedy this, further work is needed, the new understanding and the technique emerging from this study seems to be a useful tool in its own right.

2.5 Quasi-Continuum Limits (Rosenau)

Our main effort was directed to explore new possible strategies to achieve this goal of a generalized hydrodynamics. The complexity of the problem warrants starting with simple model for the germ idea to be explored. To this end we studied the model of persisting random walker as a paradigm of generalized hydrodynamics.

In the continuum limit, the unbiased, continuous time, random walk yields the diffusion equation, which may be considered as the simplest analog of the Navier-Stokes equations. Not surprisingly, the short wavelength spectrum of the diffusion equation is completely different from its discrete antecedent. These spectra cannot be expected to be the same since one dynamics is the long wavelength limit of the other.

Now consider a correlated random walk; it is a probabilistic equivalent of a colliding particle with a finite mass. Its evolution from a given site depends on its history. Such a motion leads to the telegraphers equation in the continuum (low-k) limit. This equation is free from two major difficulties of the diffusion approximation:

1) the paradox of infinite propagation speed,

2) the linear flux-gradient relations.

The idea we explored is based on the observation that the telegrapher equation approximates the original process surprisingly well for all wavelengths. We do not have the right to expect this to occur, yet it does. Even though the telegrapher equation is obtained in the long-wave length limit, it reproduces the spectrum for all wavelengths surprisingly well. In other words, a derivation which can be justified only for small gradients leads to an equation good for arbitrary gradients. This fact helps one to realize that one folly of the classical Chapman-Enskog procedure is not its expansion in small gradients, but rather that its ordering disregards inertia. With inertia intact one expects to derive a telegraphers type system with domain of validity extending beyond the formal derivation [3].

In the more general area of interest to the Air Force, we have studied interfacial instabilities in hydrodynamics. Our main effort was to develop an amplitude equation describing the evolution and rupture of thin films, separating two liquids of different density and viscosity [10].

3. Recent Articles about the Research

3.1. Articles Appearing or Accepted in Reviewed Journals

Work Specific to our Grant

[1] B. Bayly, C.D. Levermore and T. Passot, *Density Variations in Weakly Compressible Fluid Flow*, Phys. of Fluids A 4 (1992), 945–954.

[2] M. Brio and J. Hunter, *Asymptotic Equations for Conservation Laws of Mixed Type*, Wave Motion 16 (1992), 57–65.

[3] P. Rosenau, *Tempered Diffusion: A Transport Process with Propagating Fronts and Inertial Delay*, Phys. Rev. A – Rapid Comm. (1992).

[4] M. Brio and J. Hunter, *Mach Reflection for the Two Dimensional Burgers Equation*, Physica D 60 (1992), 194–214.

[5] C. Bardos, F. Golse, and C.D. Levermore, *Fluid Dynamic Limits of Kinetic Equations II: Convergence Proofs for the Boltzmann Equation*, Comm. Pure & Appl. Math. 46 (1993), 667–753.

[6] C.D. Levermore, *Entropic Convergence and the Linearized Limit for the Boltzmann Equation*, Comm. in P.D.E. 18 (1993), 1231–1248.

[7] S. Jin and C.D. Levermore, *Fully Discrete Numerical Transfer in Diffusive Regimes*, Trans. Th. & Stat. Phys. 22 (1993), 739–791.

[8] C.D. Levermore and B.A. Wagner, *Robust Fluid Dynamical Closures of the Broadwell Model*, Phys. Lett. A 174 (1993), 220–228.

[9] G.Q. Chen, C.D. Levermore and T.P. Liu, *Hyperbolic Conservation Laws with Stiff Relaxation Terms and Entropy*, Comm. Pure & Appl. Math. 47 (1994), 787–830.

[10] Y. Cheng, M. Brio, and G.M. Webb, *The Structure of Shock Waves in Asymptotic Magnetohydrodynamic System*, Canadian J. Appl. Math. Quarterly, (accepted 1995).

Work Related to the General Scientific Program of the Air Force

[11] A. Yarin, A. Oron and P. Rosenau, *Capillary Instability of Thin Liquid Film in a Cylinder*, Phys. of Fluids A (to appear 1992).

13

[12] G.M. Webb, T.I. Woodward, M. Brio and G.P. Zank, *Linear Magnetosonic N-Waves and Green's Functions*, J. Plasma Phys. **49** (1993), 465–489.

[13] C.R. Doering, J.D. Gibbon and C.D. Levermore, *Weak and Strong Solutions of the Complex Ginzburg-Landau Equation*, Physica D **71** (1994), 285–318.

[14] G.M. Webb, M. Brio and G.P. Zank, *Steady MHD Flows with an Ignorable Coordinate and the Potential Transonic Flow Equation*, J. Plasma Phys. **64** (1994), 141–173.

[15] M. Hays, C.D. Levermore, and P.D. Miller, *Macroscopic Lattice Dynamics*, Physica D **79** (1994), 1–15.

[16] G.M. Webb, M. Brio, G.P. Zank, and T. Story, *Contact and Pressure Balance Structures in Two Fluid Cosmic Ray Hydrodynamics*, Astrophys. J., (to appear April 1995).

[17] B.H. Elton, C.D. Levermore and G.H. Rodrigue, *Convergence of Convective -Diffusive Lattice Boltzmann Methods*, SIAM J. Num. Anal. (to appear October 1995).

[18] N.M. Ercolani, C.D. Levermore, and T. Zhang, *The Behavior of the Weyl Function in the Zero-Dispersion KdV Limit*, Comm. Math. Phys. (accepted 1994).

[19] P.D. Miller, N.M. Ercolani, I.M. Krichever, and C.D. Levermore, *Finite Genus Solutions to the Ablowitz-Ladik Equations*, Comm. Pure & Appl. Math. (accepted 1994).

[20] C.D. Levermore, W. Nadler, and D.L. Stein, *Random Walks on a Fluctuating Lattice: A Renormalization Group Approach Applied in One Dimension*. Phys. Rev. E (accepted 1995).

3.2. Articles Submitted to Reviewed Journals

Work Specific to our Grant

[21] S. Jin and C.D. Levermore, *Numerical Schemes for Hyperbolic Conservation Laws with Stiff Relaxation Terms*, SIAM J. Num. Anal. (submitted 1993).

[22] A. Harten, P.D. Lax, C.D. Levermore, and W.J. Morokoff, *Convex Entropies and Hyperbolicity for General Euler Equations*, SIAM J. Num. Anal. (submitted 1995).

[23] C.D. Levermore, *Moment Closure Hierarchies of the Kinetic Theories*. J. Stat. Phys. (submitted 1995).

[24] C.D. Levermore, W.J. Morokoff, and B.T. Nadiga, *Moment Realizability and the Validity of the Navier-Stokes Approximation for Rarefied Gas Dynamics*, Physics of Fluids (submitted 1995).

[25] M. Brio and J.K. Hunter, *Irregular Reflection of Weak Shocks*, Proc. V Int. Conf. Hyperbolic Problems, Stony Brook (submitted 1994).

[26] G.M. Webb and M. Brio, *Symmetries of the TDNLS Equations for Weakly Nonlinear Dispersive MHD Waves*, J. Plasma Phys. (submitted 1995).

Work Related to the General Scientific Program of the Air Force

[27] R. Camassa, D.D. Holm, and C.D. Levermore *Long-Time Shallow Water Equations with a Varying Bottom*, J. Fluid Mech. (submitted 1994).

[28] P.D. Miller, N.M. Ercolani, and C.D. Levermore, *Modulation Theory in the Presence of Phase Locking*, Physica D (submitted 1994).

[29] G. Cruz-Pacheco, C.D. Levermore, and B.P. Luce, *Complex Ginzburg-Landau Equations as Perturbations of Nonlinear Schrödinger Equations*, Physica D (submitted 1995).

3.3. Books or Book Chapters Published or to be Published

Work Specific to our Grant

[30] M. Brio and P. Rosenau, *Stability of Shock Waves for a* 3 × 3 *System of Model MHD Equations*; in "Proc. 4th Int. Conf. on Hyperbolic Problems" (Taormina, 1992), A. Donato ed., Notes on Numerical Fluid Mechanics **43** (1993), 77–83.

[31] C.D. Levermore, *An Introduction to Kinetic Theory*, in "Dynamical Systems and Probabilistic Methods for Nonlinear Waves" (AMS-SIAM Summer School, Berkeley, 20 June - 1 July 1994), P. Deift, C.D. Levermore, and E. Wayne eds., Lect. Appl. Math. ??, AMS, Providence (1995), ?? pages.

Work Related to the General Scientific Program of the Air Force

[32] C.D. Levermore, *Fluid Dynamical Limits of Discrete Kinetic Theories*; in "Macroscopic Simulations of Complex Phenomena" (NATO Advanced Study Institute, Alghero, Italy, 15-27 July 1991), M. Mareschal and B. Holian eds., NATO ASI Series B **292**, Plenum, New York (1992), 173–185.

[33] P.D. Lax, C.D. Levermore, and S. Venakides, *The Generation and Propagation of Oscillations in Dispersive Initial-Value Problems and Their Limiting Behavior* in "Important Developments in Soliton Theory", A.S. Fokas and V.E. Zakharov eds., Springer Series in Nonlinear Dynamics, Springer-Verlag, New York (1993), 205–241.

[34] S. Jin, C.D. Levermore, and D.W. McLaughlin, *The Behavior of Solutions of the NLS Equation in the Semiclassical Limit*, in "Singular Limits of Dispersive Waves", (NATO Advanced Research Workshop, Lyon, France, 8-12 July 1991), N.M. Ercolani, I.R. Gabitov, C.D. Levermore, and D. Serre eds., NATO ASI Series B **320**, Plenum, New York (1994), 173–185.

[35] C.D. Levermore and J.G. Liu, *Oscillations in Numerical Experiments*, in "Singular Limits of Dispersive Waves", (NATO Advanced Research Workshop, Lyon, France, 8-12 July 1991), N.M. Ercolani, I.R. Gabitov, C.D. Levermore, and D. Serre eds., NATO ASI Series B **320**, Plenum, New York (1994), 329–346.

[36] M. Brio and J. K. Hunter, *A von Neumann reflection for the 2-D Burgers equation*, in "Fifty Years of Computational Mathematics", W. Gautschi ed., AMS Proceeding of Symposia in Applied Mathematics, **48** (1994), 265–269.

[37] C.D. Levermore and M. Oliver, *The Complex Ginzburg-Landau Equation as a Model Problem*, in "Dynamical Systems and Probabilistic Methods for Nonlinear Waves" (AMS-SIAM Summer School, Berkeley, 20 June - 1 July 1994), P. Deift, C.D. Levermore, and E. Wayne eds., Lect. Appl. Math. ??, AMS, Providence (1995), 49 pages.

3.4. Work in Progress

Work Specific to our Grant

[38] F. Golse, S. Jin and C.D. Levermore, *The Convergence of Numerical Transfer Schemes in Diffusive Regimes*, SIAM J. Num. Anal. (to be submitted 1995).

[39] M. Brio and J. Hunter, *Numerical Study of Weak Mach Reflection near the Triple Point.* (in preparation 1995).

[40] M. Brio, J. Hunter and D. Johnson, *Canonical Equation for MHD and Elastic Waves near the Triple Umbilic Point.* (in preparation 1995).

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[41] M. Brio and P. Rosenau, *Nonstrictly Hyperbolic System of Conservation Laws with Applications to MHD Shock Waves*, Preprint, (to be submitted 1995).

[42] M. Brio and G.M. Webb, *Study of One-Way MHD Model Equation.* (in preparation 1995).

[43] C.P.T. Groth and C.D. Levermore, *Beyond the Navier-Stokes Approximation: Transport Corrections to the Gaussian Closure*, (in preparation 1995).

[44] C.D. Levermore and W.J. Morokoff, *The Gaussian Moment Closure for Gas Dynamics*, (in preparation 1995).

[45] G.M. Webb, M. Brio, G.P. Zank, and T. Story, *Wave-Wave Interaction in Two Fluid Cosmic Ray Hydrodynamics*, (in preparation 1995).

[46] G.M. Webb and M. Brio, *Prolongation Structures and Conservation Laws of the TDNLS Equation*, (in preparation 1995).

Work Related to the General Scientific Program of the Air Force

[47] S. Jin, C.D. Levermore, and D.W. McLaughlin, *The Semiclassical Limit of the Defocusing NLS Hierarchy*, Comm. Pure & Appl. Math. (to be submitted 1995).

[48] K. Horsch and C.D. Levermore, *Attractors for the Complex Ginzburg-Landau Equation in Lyapunov Cases*, Physica D (to be submitted 1995).

[49] C.D. Levermore, and J.G. Liu, *Large Oscillations Arising in a Numerical Scheme*, Physica D (to be submitted 1995).

[50] C.D. Levermore, M. Oliver, and E. Titi, *Global Well-Posedness for Certain Shallow Water Equations with a Varying Bottom*, Arch. Rat. Mech. & Anal. (to be submitted 1995).

[51] N. Ercolani, S. Jin, C.D. Levermore and W. MacEvoy, *The Zero Dispersion Limit of the NLS/mKdV Hierarchy for the Non-selfadjoint ZS Operator*, preprint (1993).

[52] C.D. Levermore, M. Oliver, and E. Titi, (in preparation 1995).

[53] C. Cercignani, I.M. Gamba, and C.D. Levermore, (in preparation 1995).

4. List of Participating Professionals

4.1. Investigators

Moysey Brio
David Levermore (PI)

4.2. Consultants Supported by Our Grant

Philip Rosenau

4.3. Graduate Students Supported by Our Grant

4.3.1. Academic Year Support

S92: Warren MacEvoy, Shan Jin, Gustavo Cruz-Pacheco.
F92: Gustavo Cruz-Pacheco, Vijayabharat Lingala.
S93: Gustavo Cruz-Pacheco, Vijayabharat Lingala.
F93: Gustavo Cruz-Pacheco.
S94: Warren MacEvoy, Gustavo Cruz-Pacheco, Karla Horsch, Peter Miller.
F94: Mark Hays, Warren MacEvoy, Peter Miller, Anita Rado.

4.3.2. Summer Support

92: Gustavo Cruz-Pacheco, Shan Jin.
93: YiFen Cheng, Gustavo Cruz-Pacheco, Vijayabharat Lingala.
94: Warren MacEvoy, Peter Miller.

4.4. Postdoctoral Associates Supported by Our Grant

None.

4.5. Other Postdoctoral Associates

92-93 Barbara A. Wagner
93-94 William J. Morokoff

5. Coupling Activities

5.1. Organizational Activities Related to the Grant

C.D. Levermore – Minisymposium Organizer, SIAM Summer Meeting, Los Angeles, 20 July 1992, "The Reemegence of Kinetic Theory in Applications".

C.D. Levermore – Workshop Organizer, Mesoscale Modeling Workshop, Mathematical Sciences Research Institute, Berkeley, 23–27 May 1994.

C.D. Levermore – Summer School Organizer, AMS-SIAM Summer School, Berkeley, 20 June – 1 July 1994, "Dynamical Systems and Probabilistic Methods for Nonlinear Waves".

C.D. Levermore – Minisymposium Organizer, SIAM Summer Meeting, San Diego, 20 July 1994, "Kinetic Theory in Applications".

C.D. Levermore – Workshop Organizer, The Institute for Advanced Study, Princeton, 27–31 March 1995, "Applied Kinetic Theory".

5.2 Presentations Related to the Grant

M. Brio, *Weak shock Mach reflection*, Problems in Computational Fluid Mechanics, Stonybrook, November 1991.

M. Brio, *Numerical Simulations of MHD Shock Waves*, 3-D PIC and MHD Workshop, Sponsored by the Air Force Office of Scientific Research, Phillips Laboratory, New Mexico, January 1992.

M. Brio, *Nonstrictly Hyperbolic System with Applications to Magnetohydrodynamics*, 4th Int. Conf. on Hyperbolic Problems, Italy, March 1992.

M. Brio, *One-Way Triple Point Model MHD Equations*, Applied Math. Working Seminar, University of Arizona, May 1992.

M. Brio, *Triple Point Model MHD Equations*, Annual Meeting of the Canadian Math. Soc., June 1992.

M. Brio, *Nonstrictly Hyperbolic System with Applications to Magnetohydrodynamics*, SIAM Annual Meeting, Los Angeles, July 1992.

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M. Brio, *Triple point model MHD equations*, Annual Meeting of the Canadian Math. Soc., August 1993.

M. Brio, *High performance computing in MHD computations*, DOE High Performance Computing Conference, Albuquerque, January 1994.

M. Brio, *Weak shock Mach reflection*, Fifth International Conference on Hyperbolic Problems, June 1994.

C.D. Levermore, *Hyperbolic Conservation Laws with Stiff Relaxation Terms and Entropy*, Seminar, Courant Institute, New York University, December 1991.

C.D. Levermore, *Hyperbolic Conservation Laws with Stiff Relaxation Terms and Entropy*, Mathematics Colloquium, Arizona State University, February 1992.

C.D. Levermore, *Hyperbolic Conservation Laws with Stiff Relaxation Terms and Entropy*, Mathematics Colloquium, University of Michigan, March 1992.

C.D. Levermore, *The Incompressible Navier-Stokes Limit for the Boltzmann Equation*, Colloquium, The Institute for Advanced Study, April 1992.

C.D. Levermore, *The Incompressible Navier-Stokes Limit for the Boltzmann Equation*, Colloquium, Duke University, April 1992.

C.D. Levermore, *Hyperbolic Conservation Laws with Stiff Relaxation Terms and Entropy*, Colloquium, Universidad Nacional Automata de Mexico, May 1992.

C.D. Levermore, *The Incompressible Navier-Stokes Limit for the Boltzmann Equation*, Colloquium, Universidad Nacional Automata de Mexico, May 1992.

C.D. Levermore, *The Incompressible Navier-Stokes Limit for the Boltzmann Equation*, Colloquium, Steklov Institute, Moscow, May 1992.

C.D. Levermore, *Robust Fluid Dynamical Closures for the Broadwell Model*, Soc. for Indust. and Appl. Math. Annual Meeting, Los Angeles, July 1992.

C.D. Levermore, *Numerical Schemes for Hyperbolic Conservation Laws with Stiff Relaxation Terms*, Ocean Modeling Workshop, Los Alamos, October 1992.

C.D. Levermore. *The Incompressible Navier-Stokes Limit for the Boltzmann Equation*, Colloquium, University of California at Irvine, October

1992.

C.D. Levermore, *Entropic Convergence and the Linearized Limit for the Boltzmann Equation*, American Math. Soc., Los Angeles, November 1992.

C.D. Levermore, *Entropic Convergence and the Linearized Limit for the Boltzmann Equation*, Colloquium, Courant Institute, New York University, November 1992.

C.D. Levermore, *The Incompressible Navier-Stokes Limit for the Boltzmann Equation*, Colloquium, Duke University, November 1992.

C.D. Levermore, *The Incompressible Navier-Stokes Limit for the Boltzmann Equation*, Colloquium, Georgia Tech, November 1992.

C.D. Levermore, *Entropic Convergence and the Linearized Limit for the Boltzmann Equation*, Mathematical Physics Seminar, University of Arizona, November 1992.

C.D. Levermore, *Entropic Convergence and the Linearized Limit for the Boltzmann Equation*, Turbulence Seminar, University of Arizona, November 1992.

C.D. Levermore, *Moment Closure Hierarchies for Kinetic Theories*, Turbulence Seminar, University of Arizona, February 1992.

C.D. Levermore, *Moment Closure Hierarchies for Kinetic Theories*, AFOSR Contractors Meeting, Washington University, St. Louis, May 1993.

C.D. Levermore, *The Incompressible Navier-Stokes Limit for the Boltzmann Equation*, Sackler Lecture, Tel Aviv University, January 1994.

C.D. Levermore, *Moment Closure Hierarchies for Kinetic Theories*, Sackler Lecture, Tel Aviv University, January 1994.

C.D. Levermore, *Moment Closure Hierarchies for Kinetic Theories*, Kinetic Theory Workshop, E.N.S. Cachan, January 1994.

C.D. Levermore, *Moment Closure Hierarchies for Kinetic Theories*, Applied Mathematics Seminar, U.C.L.A., April 1994.

C.D. Levermore, *Moment Closure Hierarchies for Kinetic Theories*, Aerospace Colloquium, University of Arizona, April 1994.

C.D. Levermore, *Moment Closure Hierarchies for Kinetic Theories*, Mesoscale Modeling Workshop, Mathematical Sciences Research Institute, May 1994.

C.D. Levermore, *Fluid Dynamical Limits for Kinetic Theories*, AMS-SIAM Summer School, Mathematical Sciences Research Institute, June 1994.

C.D. Levermore, *Moment Closure Hierarchies for Kinetic Theories*, Hyperbolic Conservation Law Workshop, Stanford University, July 1994.

C.D. Levermore, *Moment Closure Hierarchies for Kinetic Theories*, Center for Nonlinear Studies Seminar, Los Alamos National Laboratory, August 1994.

C.D. Levermore, *Moment Closure Hierarchies for Kinetic Theories*, Hyperbolic Conservation Law Meeting, Sanremo, Italy, September 1994.

C.D. Levermore, *The Incompressible Navier-Stokes Limit for the Boltzmann Equation*, Applied Math Colloquium, University of Colorado, January 1994.

C.D. Levermore, *Moment Closure Hierarchies for Kinetic Theories*, Mathematics Colloquium, Georgia Tech, December 1994.

C.D. Levermore, *Moment Closure Hierarchies for Kinetic Theories*, Aerospace Colloquium, University of Michigan, December 1994.

C.D. Levermore, *An Introduction to Kinetic Theory*, Kinetic Theory Seminar, The Institute for Advanced Study, January 1995.

C.D. Levermore, *Entropic Convergence and the Linearized Limit for the Boltzmann Equation*, Analysis Seminar, The Institute for Advanced Study, January 1995.

C.D. Levermore, *Moment Closure Hierarchies for Kinetic Theories*, Mathematical Physics Seminar, Rutgers University, February 1995.

C.D. Levermore, *Moment Closure Hierarchies for Kinetic Theories*, Applied Math Colloquium, Columbia University, February 1995.

C.D. Levermore, *Moment Closure Hierarchies for Kinetic Theories*, Applied Kinetic Theory Workshop, The Institute for Advanced Study, March 1995.

External Honors Including Major Prizes, Society Awards, etc.

P. Rosenau – Ulam Scholar, Los Alamos National Laboratory, Oct 1991 – Sept 1992.

6. Discoveries and Closing Statments

The work coming out of this effort that will have the most major impact is the derivation of a hierarchy of closed systems of moment equations corresponding to any classical kinetic theory. As was mentioned earlier, this work has, before its publication, attracted the attention of researchers a round the world. Derivative work has already been submitted to journals from the University of Michigan group of Gambosi, Groth, Roe, and Brown. Other work is forthcoming out of Bordeaux, France by a group led by Charrier. Also note the interest it has generated evidenced in Section 5 by colloqiuum and seminar invitations. Only time will tell if this work will live up to its promise, but the theory has so much structurally going for it that we will certainly learn a lot by pursuing it further.